5.14: PROBLEM DEFINITION

Situation:

In a circular duct the velocity profile is $v(r) = V_0 \left(1 - \frac{r}{R}\right)$.

Find:

Ratio of mean velocity to center line velocity, $\frac{\bar{V}}{V_0}$.

PLAN

Apply the integral form of the flow rate equation, because velocity is not constant across the cross-section.

SOLUTION

Flow rate equation

$$Q = \int v dA$$

where $dA = 2\pi r dr$. Then

$$\begin{aligned}
Q &= \int_0^R V_0 \left(1 - \left(\frac{r}{R}\right)\right) 2\pi r dr \\
&= V_0(2\pi) \left(\frac{r^2}{2} - \frac{r^3}{3R}\right) |_0^R \\
&= 2\pi V_0 \left(\frac{R^2}{2} - \frac{R^2}{3}\right) \\
&= (2/6)\pi V_0 R^2
\end{aligned}$$

Average Velocity

$$\begin{aligned}
\bar{V} &= \frac{Q}{A} \\
\frac{\bar{V}}{V_0} &= \frac{Q}{A} \frac{1}{V_0} \\
&= \frac{(2/6)\pi V_0 R^2}{\pi R^2} \frac{1}{V_0}
\end{aligned}$$

$$\boxed{\frac{\bar{V}}{V_o} = \frac{1}{3}}$$

5.31: PROBLEM DEFINITION

Situation:

A shell and tube heat exchanger with one pipe inside another pipe. Liquids flow in opposite directions.

$V_o = V_i$, $Q_o = Q_i$.

Find:

Find ratio of diameters.

PLAN

Use discharge equation $Q = AV$ and neglect pipe wall thickness.

SOLUTION

Discharge and velocity the same so

$$Q = A_{\text{inner}}V = A_{\text{outer}}V$$

Therefore

$$\frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}D_i^2$$

so

$$\boxed{\frac{D_o}{D_i} = \sqrt{2}}$$

$\boxed{\text{5.53: PROBLEM DEFINITION}}$

Situation:

Air flow downward through a pipe and then outward between two parallel disks.
$Q = 0.380\,\mathrm{m}^3/\,\mathrm{s}$, $r = 20\,\mathrm{cm}$
$D = 0.1\,\mathrm{m}$, $h = 0.6\,\mathrm{cm}$.

Find:

(a) Expression for acceleration at point A.
(b) Value of acceleration at point A.
(c) Velocity in the pipe.

PLAN

Apply the flow rate equation.

SOLUTION

---

a)
Flow rate equation

$$V_r = \frac{Q}{A} = \frac{Q}{2\pi r h}$$

Evaluate convective acceleration along a radial pathline ($s \to r$)

$$\begin{aligned} a_c &= \frac{V_r \partial V_r}{\partial r} \\ &= \left(\frac{Q}{2\pi r h}\right)(-1)\left(\frac{Q}{2\pi r^2 h}\right) \\ & \boxed{a_c = \frac{-Q^2}{r(2\pi r h)^2}} \end{aligned}$$

---

b)

$$\begin{aligned} V_{\text{pipe}} &= \frac{Q}{A_{\text{pipe}}} \\ &= \frac{(0.380\,\mathrm{m}^3/\,\mathrm{s})}{\frac{\pi}{4}\,(0.1\,\mathrm{m})^2} \\ & \boxed{V_{\text{pipe}} = 48.4\ \mathrm{m/s}} \end{aligned}$$

---

c)

$$\begin{aligned} a_c &= -\frac{(0.38\,\mathrm{m}^3/\,\mathrm{s})^2}{(0.2\,\mathrm{m})(2\pi\,(0.2\,\mathrm{m})\,(0.006\,\mathrm{m}))^2} \\ & \boxed{a_c = -12,700\ \mathrm{m/s}^2} \end{aligned}$$

5.65: PROBLEM DEFINITION

Situation:

Gas flows in a round conduit that tapers to a smaller diameter.
$D_1 = 1.2\,\mathrm{m}$, $D_2 = 0.6\,\mathrm{m}$, $V_1 = 15\,\mathrm{m/s}$.

Find:

Mean velocity at section 2.

Properties:

$\rho_1 = 2.0\,\mathrm{kg/m^3}$, $\rho_2 = 1.5\,\mathrm{kg/m^3}$.

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$
\begin{aligned}
V_2 &= \frac{\rho_1 A_1 V_1}{\rho_2 A_2} \\
&= \frac{\rho_1 D_1^2 V_1}{\rho_2 D_2^2} \\
&= \frac{(2.0\,\mathrm{kg/m^3})\,(1.2\,\mathrm{m})^2\,(15\,\mathrm{m/s})}{(1.5\,\mathrm{kg/m^3})\,(0.6\,\mathrm{m})^2} \\
& \boxed{V_2 = 80.0\ \mathrm{m/s}}
\end{aligned}
$$

5.73: PROBLEM DEFINITION

Situation:

O_2 and CH_4 are mixed in a mixer before exiting.
$V_{O_2} = V_{CH_4} = 5\,\mathrm{m/s}$.
$A_{CH_4} = 1\,\mathrm{cm}^2$, $A_{O_2} = 3\,\mathrm{cm}^2$.

Find:

Exit velocity of the gas mixture, V_e.

Properties:

From Table A.2: $R_{O_2} = 260\,\mathrm{J/\,kg\,K}$, $R_{CH_4} = 518\,\mathrm{J/\,kg\,K}$.
$T = 100\,^\circ\mathrm{C}$, $\rho = 1.9\,\mathrm{kg/\,m^3}$, $p = 200\,\mathrm{kPa}$.

PLAN

Apply the ideal gas law to find inlet density. Then apply the continuity equation.

SOLUTION

Ideal gas law

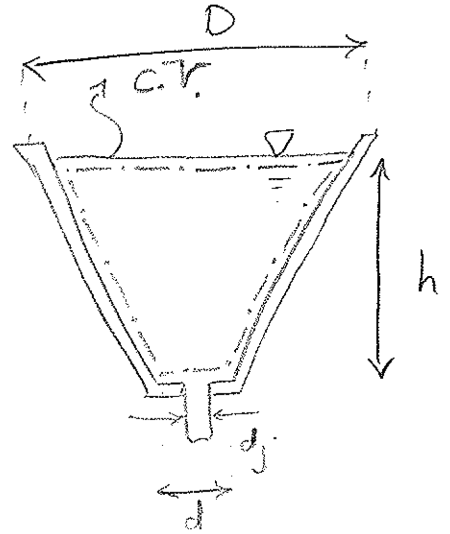
$$\begin{aligned}
\rho_{0_2} &= \frac{p}{RT} \\
&= \frac{200,000\,\mathrm{Pa}}{(260\,\mathrm{J/\,kg\,K})\,(273+100)\ \mathrm{K}} \\
&= 2.06\ \mathrm{kg/m^3} \\
\rho_{CH_4} &= \frac{200,000\,\mathrm{Pa}}{(518\,\mathrm{J/\,kg\,K})\,(273+100)\ \mathrm{K}} \\
&= 1.03\ \mathrm{kg/m^3}
\end{aligned}$$

Continuity equation

$$\begin{aligned}
\sum \dot{m}_i &= \sum \dot{m}_o \\
\rho_\mathrm{e} V_\mathrm{e} A_\mathrm{e} &= \rho_{\mathrm{O_2}} V_{\mathrm{O_2}} A_{\mathrm{O_2}} + \rho_{\mathrm{CH_4}} V_{\mathrm{CH_4}} A_{\mathrm{CH_4}} \\
V_\mathrm{e} &= \frac{2.06\,\mathrm{kg/\,m^3} \times 5\,\mathrm{m/\,s} \times 3\,\mathrm{cm^2} + 1.03\,\mathrm{kg/\,m^3} \times 5\,\mathrm{m/\,s} \times 1\,\mathrm{cm^2}}{1.9\,\mathrm{kg/\,m^3} \times 3\,\mathrm{cm^2}} \\
& \boxed{V_\mathrm{e} = 6.33\ \mathrm{m/s}}
\end{aligned}$$

Problem (5.83)

Solution 1



Find

Derive a formula for the time to drain.

Solution

The continuity Eq. for the control volume, shown in the figure, is:

$$\frac{d}{dt}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} = 0 \qquad (1)$$

In this problem, ρ is constant, and there is only one outflow through the control surface. The velocity of the water is also assumed to be uniform across the jet just as it leaves the tank. Thus, Eq. (1) can be simplified to:

$$\frac{d\forall}{dt} = -Q_e = -A_j V_e, \qquad (2)$$

where $V_e = \sqrt{2gh}$ based on Bernoulli Eq.

The volume of the tank as a function of D, d, h can be expressed as:

$$\forall = \frac{\pi}{12}\left(D^2 + d^2 + Dd\right)h \qquad (3)$$

Substituting D with $d + c_1 h$ is Eq. (3) yields to:

$$\forall = \frac{\pi}{12}\left[(d+c_1h)^2 + d^2 + d(d+c_1h)\right]h = \frac{\pi}{12}\left(3d^2h + 3dc_1h^2 + c_1^2h^3\right) \qquad (4)$$

So $\frac{d\forall}{dt}$ can be written as:

$$\frac{d\forall}{dt} = \frac{d\forall}{dh}\frac{dh}{dt} = \frac{\pi}{4}\left(d^2 + 2dc_1h + c_1^2h^2\right)\frac{dh}{dt} \qquad (5)$$

From Eqs. (2) & (5) :

$$\frac{\pi}{4}\left(d^2 + 2dc_1h + c_1^2h^2\right)\frac{dh}{dt} = -\frac{\pi}{4}d_j^2\sqrt{2gh}$$

$$\Rightarrow dt = -\frac{1}{d_j^2\sqrt{2g}}\left(d^2h^{-\frac{1}{2}} + 2dc_1h^{\frac{1}{2}} + c_1^2h^{\frac{3}{2}}\right) \qquad (6)$$

Eq. (6) can be integrated to find the time of fall of liquid surface from $h = h_o$ to $h = h$:

$$\int_0^t dt = \frac{-1}{d_j^2\sqrt{2g}}\int_{h_o}^{h}\left(d^2h^{-\frac{1}{2}} + 2dc_1h^{\frac{1}{2}} + c_1^2h^{\frac{3}{2}}\right)dh$$

$$\Rightarrow t = \frac{-1}{d_j^2\sqrt{2g}}\left(2d^2h^{\frac{1}{2}} + \frac{4}{3}dc_1h^{\frac{3}{2}} + \frac{2}{5}c_1^2h^{\frac{5}{2}}\right)\Bigg|_{h_o}^{h}$$

$$\Rightarrow \boxed{t = \frac{-1}{d_j^2\sqrt{2g}}\left[2d^2\left(h^{\frac{1}{2}} - h_o^{\frac{1}{2}}\right) + \frac{4}{3}dc_1\left(h^{\frac{3}{2}} - h_o^{\frac{3}{2}}\right) + \frac{2}{5}c_1^2\left(h^{\frac{5}{2}} - h_o^{\frac{5}{2}}\right)\right] \qquad (7)}$$

Now, we can insert values of h_0, h, d, C_1 and d_j in Eq. (7):

$(h_0 = 1\,m,\ h = 20\,cm,\ d = 20\,cm,\ C_1 = 0.3,\ d_j = 5\,cm)$

$$t = \frac{+1}{(0.05m)^2 (2 \times 9.8\, m/s^2)^{\frac{1}{2}}} \Big[2(0.2m)^2 \left((1m)^{\frac{1}{2}} - (0.2m)^{\frac{1}{2}} \right)$$

$$+ \frac{4}{3}(0.2m)\,0.3 \left((1m)^{\frac{3}{2}} - (0.2m)^{3/2} \right) + \frac{2}{5}\,0.3^2 \left((1m)^{\frac{5}{2}} - (0.2m)^{\frac{5}{2}} \right) \Big]$$

$$\Longrightarrow \boxed{t \simeq 13.8\,s}$$

Problem (5.83) $\boxed{\text{Solution 2}}$

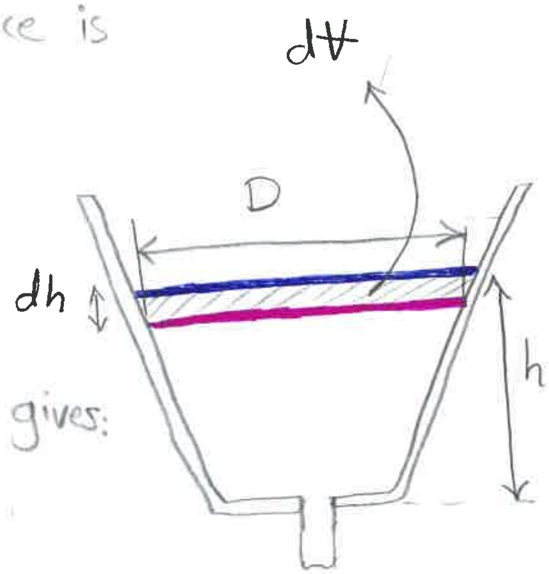
In the first solution of the problem, we found that the continuity Eq. for this tank can be simplified to:

$$\frac{d\forall}{dt} = -Q_e = -V_e A_j , \qquad (1)$$

where $\forall$ is the volume of the liquid in the tank, and Q_e is the discharge at the bottom of the tank.

In order to find $\frac{d\forall}{dt}$ in the 1st solution, we wrote $\forall$ as a function of h, and then, we calculated $\frac{d\forall}{dt}$ using the chain rule which expresses $\frac{d\forall}{dt} = \frac{d\forall}{dh} \cdot \frac{dh}{dt}$. However, we use another method in this solution to find $\frac{d\forall}{dt}$. In the below figure, the liquid surface at $t=t$ is shown with the blue color, whereas its position at $t = t+dt$ is shown with the red color. At this infinitesimal time (dt), the change in the diameter of the liquid surface is negligible (i.e., $dD \ll D$), so $d\forall$ can be assumed as a cylinder:

$$d\forall = \frac{\pi}{4} D^2 dh$$



Deviding two sides of above equation by dt gives:

$$\frac{d\forall}{dt} = \frac{\pi}{4} D^2 \frac{dh}{dt} \qquad (2)$$

From Eqs. (1) & (2):

$$D^2 \frac{dh}{dt} = -D_j^2 \sqrt{2gh} \qquad (3)$$

Substituting D with $d + C_1 h$ in Eq. (3) gives:

$$(d^2 + 2dC_1 h + C_1^2 h^2)\frac{dh}{dt} = -D_j^2 \sqrt{2gh} \qquad (4)$$

So dt can be written as:

$$dt = -\frac{(d^2 + 2dC_1 h + C_1^2 h^2)\,dh}{\sqrt{2g}\, h^{\frac{1}{2}}\, d_j^2} \qquad (5)$$

Integrating from Eq. (5) gives:

$$t = -\int_{h_0}^{h} \frac{(d^2 + 2dC_1 h + C_1^2 h^2)\,dh}{\sqrt{2g}\, h^{\frac{1}{2}}\, d_j^2}$$

$$t = \frac{2}{d_j^2 \sqrt{2g}} \left[d^2 h^{1/2} + \frac{2}{3} d C_1 h^{\frac{3}{2}} + \frac{1}{5} C_1^2 h^{\frac{5}{2}} \right]_h^{h_0} \qquad (6)$$

Evaluating the limits of integration gives:

$$\boxed{t = \frac{2}{d_j^2 \sqrt{2g}} \left[d^2 (h_0^{\frac{1}{2}} - h^{\frac{1}{2}}) + \frac{2}{3} d C_1 (h_0^{\frac{3}{2}} - h^{\frac{3}{2}}) + \frac{1}{5} C_1^2 (h_0^{\frac{5}{2}} - h^{\frac{5}{2}}) \right] \qquad (7)}$$

Eq. (7) is exactly the same as the one obtaind in the first solution. It, therefore, again confirms that the two solutions can be interchangeably used. However, the second solution is more suitable for the geometries whose volumes are difficult to calculate (e.g., P5.78 in 9th Ed. and P5.84 in 10th Ed.)

5.90: PROBLEM DEFINITION

Situation:

A piston moves in a cylinder and drives exhaust gas out an exhaust port in a four cycle engine.

$\dot{m} = \frac{0.65 p_c A_v}{\sqrt{RT_c}}$, $d_{bore} = 0.1\,\text{m}$.

$L = 0.1\,\text{m}$, $A_v = 1\,\text{cm}^2$.

$V = 30\,\text{m/s}$.

Find:

Rate at which the gas density is changing in the cylinder.

Assumptions:

The gas in the cylinder is ideal and has a uniform density and pressure.

Properties:

$T = 600\,°\text{C}$, $R = 350\,\text{J/kg K}$, $p = 300\,\text{kPa}$.

SOLUTION

Continuity equation. Control volume is defined by piston and cylinder.

$$\begin{aligned}
\frac{d}{dt}(\rho V) + \frac{0.65 p_c A_v}{\sqrt{RT_c}} &= 0 \\
V\frac{d\rho}{dt} + \rho\frac{dV}{dt} + \frac{0.65 p_c A_v}{\sqrt{RT_c}} &= 0 \\
\frac{d\rho}{dt} &= -\left(\frac{\rho}{V}\right)\frac{dV}{dt} - \frac{0.65 p_c A_v}{V\sqrt{RT_c}} \\
V &= (\pi/4)(0.1\,\text{m})^2(0.1\,\text{m}) = 7.854 \times 10^{-4}\ \text{m}^3 \\
\frac{dV}{dt} &= -(\pi/4)(0.1\,\text{m})^2(30\,\text{m/s}) = -0.2356\ \text{m}^3/\text{s} \\
\rho &= \frac{p}{RT} = \frac{300,000\,\text{Pa}}{(350\,\text{J/kg K} \times 873\,\text{K})} \\
&= 0.982\ \text{kg/m}^3 \\
\frac{d\rho}{dt} &= -\frac{0.982\,\text{kg/m}^3}{7.854 \times 10^{-4}\,\text{m}^3} \times (-0.2356\,\text{m}^3/\text{s}) \\
&\quad -\frac{0.65 \times 300,000\,\text{Pa} \times 1 \times 10^{-4}\,\text{m}^2}{7.854 \times 10^{-4}\,\text{m}^3 \times \sqrt{350\,\text{J/kg/K} \times 873\,\text{K}}} \\
&\boxed{\frac{d\rho}{dt} = 250\ \text{kg/m}^3\!\cdot\text{s}}
\end{aligned}$$

Problem 5.98

From continuity equation:

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 (A_1 / A_2)$$

$$= (24\ m/s)(2)$$

$$= 48\ \frac{m}{s}$$

From Bernoulli equation:

$$P_1 + \rho g z_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g z_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho (V_2^2 - V_1^2) + \rho g (z_2 - z_1)$$

$$P_1 - P_2 = \frac{1}{2}(1\ kg/m^3)\left[(48\ m/s)^2 - (24\ m/s)^2\right]$$

$$+ (1\ kg/m^3)(9.8\ m/s^2)(3\ m)$$

$$= 893.4\ \frac{N}{m^2}$$

Manometer equation: (Note: Air density is neglected with respect to the liquid density)

$$P_1 - P_2 = \Delta h \, \gamma_{liquid}$$

$$893.4 \ \frac{N}{m^2} = \Delta h \left(18900 \ \frac{N}{m^3} \right)$$

$$\boxed{\Delta h = 0.047 \ m}$$